

On the Apparent Superluminal Motion of a Damped Gaussian Pulse

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Alicki has demonstrated that a travelling Gaussian pulse subject to damping is indistinguishable from an undamped pulse moving with greater speed; such an effect could create the illusion of a pulse moving faster than light. In this note, an alternative derivation of the same result is presented. However, it is unlikely that this particular illusion could explain the superluminal neutrino-velocities reported by OPERA.

The recent claim that the measured velocity of mu neutrinos exceeds that of light [1] has rekindled interest in the old subject of illusory superluminal motion. Of the various proposals advanced to explain the OPERA results without jeopardy to established physics [2], that of Alicki [3] is among the most intriguing. Alicki argues that a stationary observer cannot tell a moving normal distribution subject to global exponential damping from an undamped normal distribution with different characteristic parameters, including higher (even apparently superluminal) velocity. This result, which may be of some importance in optics and other fields unrelated to neutrino physics, is re-derived in the present note by very elementary means. Unfortunately (or otherwise, if one wishes to break the light barrier in fact rather than appearance), such an effect seems unlikely to be the solution to the OPERA puzzle: the CNGS-beam pulse lacks the “fat tail” needed for the argument to apply.

To derive Alicki’s result, it is first helpful to consider the case of a Gaussian pulse with no damping at all. Let $\rho(x, t)$ denote the intensity, assumed non-negative, of a signal moving at constant speed $v > 0$ in one dimension without dispersion or distortion toward an observer at some fixed location $x = L$. By hypothesis, observer and source are in a state of mutual rest. Let the profile of the signal as initially transmitted be Gaussian with maximum height ρ_0 and full width σ at half maximum. Since transmission is perfect, the observer at L will see:

$$\rho(t) = \rho_0 \exp -\frac{(L - vt)^2}{2\sigma^2}$$

The time derivative of ρ is:

$$\dot{\rho} = \rho \frac{v}{\sigma^2} (L - vt)$$

so the observer will notice a peak at time

$$t = L/v$$

Prior to that time, ρ is increasing; afterwards, decreasing. Knowing L and measuring t , the observer easily determines the velocity of the pulse to be v , as expected.

Now introduce a simple damping:

$$\rho(t) = (\rho_0 \exp -\frac{(L - vt)^2}{2\sigma^2}) \exp -\gamma t \quad (1)$$

where γ is a positive constant. The time derivative of ρ becomes:

$$\dot{\rho} = \rho \frac{v}{\sigma^2} (L - vt - \delta L)$$

where

$$\delta L \equiv \frac{\gamma \sigma^2}{v} \geq 0 \quad (2)$$

The observer will now notice a peak at time

$$t = T \equiv (L - \delta L)/v$$

Prior to that time, ρ is increasing; afterwards, decreasing. It is therefore reasonable to define

$$v' \equiv L/T = \frac{Lv}{L - \delta L} \quad (3)$$

Note that $v' \geq v$ provided $0 \leq \delta L \leq L$.

To shew that v' could in fact be interpreted as the velocity of a moving, undamped Gaussian pulse, note that according to Eq. (2):

$$\sigma^2 \gamma t = vt \delta L \quad (4)$$

Therefore, we may rewrite Eq. (1) in terms of δL as:

$$\rho(t) = \rho_0 \exp -\frac{F}{2\sigma^2} \quad (5)$$

where

$$F \equiv L^2 + (vt)^2 - 2(L - \delta L)vt \quad (6)$$

Furthermore, define two new constants

$$\sigma' \equiv \frac{\sigma}{1 - (\delta L/L)} \quad (7)$$

and

$$\rho'_0 \equiv \rho_0 \exp -\frac{\delta L}{2\sigma^2}(2L - \delta L) \quad (8)$$

With the help of Eqs. (2), (3), (6), (7), and (8), Eq. (5) becomes:

$$\rho(t) = \rho'_0 \exp -\frac{(L - v't)^2}{2\sigma'^2} \quad (9)$$

quod erat demonstrandum.

Less formally, the Alicki effect can be understood in terms of the damping acting on the signal over time. At $t = 0$, the value of ρ in the neighbourhood of L is $\rho_0 \exp -(L/\sigma)^2/2$. A few moments later, the part of the distribution originally just a bit to the left of L will pass the observer; the damping force will have acted upon it briefly, so its magnitude will be somewhat diminished. By contrast, the regions of the distribution which at $t = 0$ were far to the left, though they may have had initially large values of ρ , will arrive at the observer much later, and the exponential decay induced by the damping force will have acted on them for a long time. Thus, the magnitude of parts of a sufficiently wide Gaussian may be greatly diminished by even a small damping, and the peak of the received signal will be shifted to the right relative to the peak in the signal as originally sent.

The constraint $\delta L \leq L$ for positive v' can also be given a qualitative interpretation. Consider the three factors which define δL according to Eq. (2). Should the damping constant γ be too large, or the initial velocity v too small, the magnitude of the signal reaching the observer will diminish from the very first observation at $t = 0$. The observer might conclude from this that the pulse is moving to the left (in agreement with Eq. (3) for this case), and that only its receding tail is being observed. The same is true, if perhaps less self-evidently, for a very broad pulse: it rises too slowly to keep up with the decay, and may also be construed as moving leftwards.

Since both source and observer are stationary in a common frame of reference, the initial velocity v may be given any value up to c without affecting the forgoing analysis. In the case $v = c$ with $\delta L \leq L$, the apparent velocity seen by the observer will always be superluminal and may even be infinite. However, no information can be transmitted faster than light in this way. Although the peak of the distribution seen by the observer arrives much sooner than it would have in the absence of damping, its magnitude cannot exceed whatever value the undamped signal would have had at that moment. Any message communicated by the arrival of the undamped peak could therefore have been transmitted just as swiftly (or more so) by specifying a particular magnitude of signal intensity as the communication threshold.

In the real world, no signal is truly Gaussian: the intensity at the origin does not begin to rise above zero in the infinitely distant past. Rather, the sender of the signal presses a button at some time $t_0 < 0$, generating output which rises in approximately Gaussian fashion to a peak at $t = 0$ and then dies away. The first evidence of this signal – the start of the long but finite rising tail – reaches the observer no sooner than some minimum time $\tau \equiv t_0 + (L/v)$. Even though

the peak of the received signal may occur at a time less than L/v , no bit of information can arrive before $t = \tau$, and thus the message “the button has been pressed” cannot be transmitted with speed greater than v .

Alicki, of course, has never claimed that his effect can be used to send information faster than light, or that it is more than an interesting illusion. He has, however, suggested that it might explain the OPERA superluminal neutrino data. This seems to me to be unlikely. The pulses from the CNGS beam used in the experiment were not Gaussian: indeed, to a first approximation, they were almost square. They entirely lacked the characteristic long tail of the normal distribution, the envelope above which the Alicki peak cannot rise.

By the “arrival” of the neutrino pulse, the OPERA group meant the arrival of the nearly vertical front, not that of the centre of the broad, relatively flat peak. It is true that a series of arguably bell-shaped sub-pulses can be discerned in the flat region, and, in the second version of his paper [4], Alicki seems to treat these, rather than the whole pulse, as the Gaussian signals which have been shifted. This cannot, however, explain the superluminal motion of the square base-pulse above which the sub-peaks rise.

Henri [5] has described a superluminal illusion involving logistic rather than Gaussian signals, clearly a better approximation to the actual shape of the CNGS-beam distribution-function. Broda [6] has suggested a general class of statistical models which may generate such illusions, given a particular method of maximum-likelihood estimation. The analysis of these more complicated proposals is beyond the scope of the present note.

Although it does not resolve the OPERA puzzle, the Alicki effect is an interesting superluminal illusion, and one which experimentalists should be able to demonstrate in the laboratory with acoustical or optical signals.

REFERENCES

1. T. Adam et al., “Measurement of the neutrino velocity with the OPERA detector in the CNGS beam” arXiv:1109.4897
2. A partial list may be found at the Net Advance of Physics website:
<http://web.mit.edu/redingtn/www/netadv/ftlNuSlow.html>
3. R. Alicki, “A possible statistical mechanism of anomalous neutrino velocity in OPERA experiment?” (version 1) arXiv:1109.5727v1
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6. B. Broda, “An OPERA inspired classical model reproducing superluminal velocities” arXiv:1110.0644v3